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General relativistic MHD simulations of monopole magnetospheres of black holes

S.S.Komissarov
Department of Applied Mathematics, The University of Leeds, Leeds LS2 9JT

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ABSTRACT

In this paper we report the results of the first ever time-dependent general relativistic magnetohydrodynamic simulations of the magnetically dominated monopole magnetospheres of black holes. It is found that the numerical solution evolves towards a stable steady-state solution which is very close to the corresponding force-free solution found by Blandford and Znajek. Contrary to the recent claims, the particle inertia does not become dynamically important near the event horizon and the force-free approximation provides a proper framework for magnetically dominated magnetospheres of black holes. For the first time, our numerical simulations show the development of an ultra-relativistic particle wind from a rotating black hole. However, the flow remains Poynting dominated all the way up to the fast critical point. This suggests that the details of the so-called "astrophysical load", where the electromagnetic energy is transferred to particles, may have no effect on the efficiency of the Blandford-Znajek mechanism.

Key words: black hole physics – magnetic fields – methods:numerical.

1 INTRODUCTION

It is now widely believed that the relativistic jets generated in active galactic nuclei, galactic microquasars and, presumably, during gamma ray bursts are powered by rapidly rotating black holes. This paradigm is largely based on the theoretical results of Blandford and Znajek (1977), who argued that, under typical conditions of astrophysical black holes, their rotational energy can be efficiently extracted in the form of magnetically dominated relativistic wind.

In fact, the original model of Blandford and Znajek was based on the approximation of steady-state force-free degenerate electrodynamics (FFDE, see also Macdonald & Thorne 1982). In this approximation the dynamical role of magnetospheric plasma is reduced to providing perfect conductivity in the space surrounding the black hole, and the particle inertia is totally neglected. Not all mathematical and physical aspects of this electrodynamic model are well understood. Similar classical systems, e.g. the Faraday disc, involve the poloidal current driven over the surface of a rotating conductor by a non-electrostatic electromotive force. Moreover, the inertia of this conductor is a key factor in the origin of this force. Obviously, there is no such conductor in the Blandford-Znajek model. Often, this missing element of the model is artificially introduced in the form of the so-called "membrane", or the "stretched horizon", located somewhat above the real event horizon (Macdonald & Thorne 1982; Thorne et al.1986). Mathematically, this is consistent with the so-called "horizon boundary conditions" employed in the Blandford-Znajek model (Znajek 1977).

However, the "Membrane paradigm" cannot always provide correct insights into the black hole electrodynamics (this would be a great mystery if it did.) Since the membrane is very close to the real horizon, the outgoing fast waves emitted by the membrane are highly red-shifted. Moreover, the membrane cannot at all communicate with the outer space by means of Alfvén waves because the inner Alfvén surface is quite far away from the horizon. As the result, these waves cannot transport angular momentum and adjust the poloidal electric current of the outgoing wind as they do so, for example, in stellar winds of magnetised stars. Punsly and Coroniti (1990) used these causality arguments to expose the lack of physical clarity in the electrodynamic model of Blandford and Znajek (1977) and its representation in the membrane paradigm. In fact, they concluded that there is no physically meaningful electromotive force in the Blandford-Znajek model but only the artificial one which has been effectively introduced via Znajek's horizon condition. If so, then the Blandford-Znajek model and, hence, its steady-state solutions have to be nonphysical. Since these FFDE solutions are, in fact, proper mathematical solutions their nonphysical nature has to give itself away via instability (Punsly & Coroniti 1990a; Punsly 2001).

In fact, more or less the same criticism was applied to

the GRMHD model of black hole magnetospheres developed by Phinney (1982) as it also utilized Znajek's horizon condition. In addition to this criticism, Punsly and Coroniti developed a rather attractive alternative GRMHD theory of black hole magnetospheres where particle inertia played the key role in the extraction of rotational energy of black holes (Punsly & Coroniti 1990b; Punsly 2001). While almost everywhere in their models the magnetic field was completely dominating, in some regions within the black hole ergosphere the plasma particles were accelerated to such a high Lorentz factor that their inertia could no longer be ignored. This led to strong inertially driven electric currents flowing across the magnetic field lines, so that the system resembled the famous Faraday disc rather closely.

In spite of the great astrophysical importance of the electrodynamic/MHD mechanism of extraction of rotational energy of black holes and admirable efforts of B.Punsly, the dramatic clash between the Blandford-Znajek and Punsly-Coroniti models seemed to remain largely unnoticed by the astrophysical community for a whole decade. Only very recently, following the developments in the theory of force-free electrodynamics (Uchida 1997; Gruzinov1999; Komissarov 2002) and the impressive progress in numerical methods for relativistic astrophysics, e.g. (Pons et al.1998; Komissarov 1999; Koide et al.1999; Koide 2003; Koldoba et al.2002; Gammie et al.2003; Del Zanna et al.2003; De Villiers & Hawley 2003), things began to change. One of most interesting recent results is concerned with the conjecture of Punsly and Coroniti on instability of the Blandford-Znajek monopole solution. In (Komissarov 2001b) this conjecture was tested by means of time-dependent general relativistic FFDE simulations and was found to be incorrect. Contrary to the appealing Punsly-Coroniti causality arguments, this steady-state monopole solution of Blandford and Znajek is asymptotically stable. There is, however, the question of validity of the FFDE approximation which has to be fully investigated before reaching any final conclusion.

There are many examples in physics and astrophysics where particular approximations fail to describe certain natural phenomena because one or another physical factor ignored in the approximation becomes important. These examples taught us always to check the limits of applicability of our theoretical models and never be satisfied by their mere self-consistency. The very relevant example can be found in the theory of magnetically dominated pulsar winds where particle inertia becomes an important factor as a result of acceleration by electromagnetic field (Mestel 1999, Section 13.2.3) Other potential examples in the theory of black hole magnetospheres are discussed by Punsly (2001, Sections 8 and 9). Recently, Punsly (2004) argued that FFDE is deficient near the event horizon where "the plasma attains an infinite inertia in a global sense". Macdonald and Thorne (1982) also anticipated a breakdown of the FFDE approximation near the event horizon, though no detailed explanations were given. Thus, in the problem of electrodynamic/MHD mechanism of extraction of rotational energy of black holes we have to verify that the particle inertia can indeed be ignored everywhere without oversimplifying this problem. This can be done via GRMHD simulations of black hole magnetospheres.

The main goal of this paper is to explore the role of particle inertia in the pair-filled monopole magnetospheres of black holes by means of such simulations. In particular, we need to know whether incorporation of initially small particle inertia will eventually lead to strong deviations from the corresponding electrodynamic solution both locally, e.g. near the event horizon, and globally. By latter we mean changes of the global system of electric current, the angular velocity of magnetic field lines, and, hence, the efficiency of the energy extraction. Since, at present only codes for the perfect relativistic MHD are readily available, the monopole magnetic configuration appears to be the most suitable one for this purpose. Indeed, this configuration allows a dissipation free FFDE solution (Blandford & Znajek 1977; Komissarov 2001b). Dipolar magnetospheres, like the one considered by Koide (2003), are less suitable because the corresponding electrodynamic solution involves a strong current sheet located within the ergosphere (Komissarov 2002; Komissarov 2004). This indicates that the approximation of perfect MHD is also likely to break down in this configuration.

2 BASIC EQUATIONS AND NUMERICAL METHOD

In this study we use the Kerr-Schild coordinates, which are more suitable for our purpose than the more popular Boyer-Lindquist coordinates as they do not introduce a coordinate singularity at the event horizon. Not only this allows to increase efficiency of computer simulations but we may also place the inner boundary of our computational domain inside the event horizon. At such a boundary we may confidently impose usual radiative boundary conditions. Thus, the disputed Znajek's horizon conditions are not involved in any form.

Using the standard notation of the 3+1 approach, the metric form of the Kerr-Schild coordinates, t, ϕ, r, θ, is

$$ds^2 = (\beta^2 - \alpha^2)dpt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j, \tag{1}$$

where γ_{ij} is the metric tensor of space,

$$\gamma_{ij} = \begin{pmatrix} \Sigma \sin^2\theta/\kappa^2 & -a\sin^2\theta(1+Z) & 0 \\ -a\sin^2\theta(1+Z) & 1+Z & 0 \\ 0 & 0 & \kappa^2 \end{pmatrix}, \tag{2}$$

α is the lapse function,

$$\alpha = 1/\sqrt{1+Z}, \tag{3}$$

and $\boldsymbol{\beta}$ is the shift vector,

$$\beta^i = (0, \frac{Z}{1+Z}, 0). \tag{4}$$

In these equations

$$\begin{aligned} \kappa^2 &= r^2 + a^2\cos^2\theta, \\ Z &= 2r/\kappa^2, \\ \Sigma &= (r^2+a^2)^2 - a^2\Delta\sin^2\theta, \\ \Delta &= r^2 + a^2 - 2r. \end{aligned}$$

and we assume that indexes 1,2,3 correspond to the ϕ-, r-, and θ-coordinates respectively. Notice that we utilize such units that $G = M = c = 1$ and 4π does not appear in the Maxwell equations.

The system of perfect general relativistic magnetohydrodynamics (GRMHD) includes the continuity equation,

$$\partial_t(\sqrt{\gamma}\rho u^t) + \partial_i(\sqrt{\gamma}\rho u^i) = 0, \tag{5}$$

the energy-momentum equations,

$$\partial_t(\sqrt{\gamma}T^t{}_\nu) + \partial_i(\sqrt{\gamma}T^i{}_\nu) = \frac{1}{2}\partial_\nu(g_{\alpha\beta})T^{\alpha\beta}\sqrt{\gamma}, \tag{6}$$

and the induction equation,

$$\partial_t(B^i) + e^{ijk}\partial_j(E_k) = 0. \tag{7}$$

Here $g_{\alpha\beta}$ is the metric tensor of spacetime, $\gamma = \det(\gamma_{ij})$, e^{ijk} is the Levi-Civita pseudo-tensor of space, ρ is the proper mass density of plasma, u^ν is its four-velocity vector. The total stress-energy-momentum tensor, $T^{\mu\nu}$, is a sum of the stress-energy momentum tensor of matter,

$$T^{\mu\nu}_{(m)} = wu^\mu u^\nu - pg^{\mu\nu}, \tag{8}$$

where p is the thermodynamic pressure and w is the enthalpy per unit volume, and the stress-energy momentum tensor of electromagnetic field,

$$T^{\mu\nu}_{(e)} = F^{\mu\gamma}F^\nu{}_\gamma - \frac{1}{4}(F^{\alpha\beta}F_{\alpha\beta})g^{\mu\nu}, \tag{9}$$

where $F^{\nu\mu}$ is the Maxwell tensor of the electromagnetic field. The electric field, $\boldsymbol{E}$, and the magnetic field, $\boldsymbol{B}$, are defined via

$$E_i = \frac{\alpha}{2}e_{ijk}\,{}^*\!F^{jk}, \tag{10}$$

and

$$B^i = \alpha\,{}^*\!F^{it}, \tag{11}$$

where ${}^*\!F^{\mu\nu}$ is the Faraday tensor of the electromagnetic field, which is simply dual to the Maxwell tensor. In the limit of ideal MHD

$$\boldsymbol{E} = -\boldsymbol{v}\times\boldsymbol{B} \quad \text{or} \quad E_i = e_{ijk}v^jB^k, \tag{12}$$

where $v^i = u^i/u^t$ is the usual 3-velocity of plasma. All the components of vectors and tensors appearing in equations (5,6,7) are the components in the basis of global coordinates.

Our numerical scheme is a generalization of the scheme for the special relativistic MHD described in Komissarov(1999), where the magnetic field is evolved using the method of constraint transport (Evans & Hawley 1988). It shares many common features with other existing numerical schemes for GRMHD and for this reason we only briefly outline its design.

This is a 2-dimensional scheme with enforced axial symmetry. The set of primitive variables include P, ρ, the components of magnetic field, $B^{\hat{i}}$, and the components of fluid four-velocity, $u^{\hat{i}}$, as measured in the orthonormal basis of the local fiducial observer (FIDO), $\{\boldsymbol{i}_{\hat{t}}, \boldsymbol{i}_{\hat{\phi}}, \boldsymbol{i}_{\hat{r}}, \boldsymbol{i}_{\hat{\theta}}\}$. The transformation from the coordinate basis to the orthonormal basis of FIDO is determined by

$$\begin{aligned}
\boldsymbol{i}_{\hat{t}} &= \frac{1}{\alpha}\frac{\partial}{\partial t} - \frac{\beta^r}{\alpha}\frac{\partial}{\partial r}, \\
\boldsymbol{i}_{\hat{\phi}} &= \frac{1}{\sqrt{\gamma_{\phi\phi}}}\frac{\partial}{\partial\phi}, \\
\boldsymbol{i}_{\hat{r}} &= \mathcal{A}\left(\frac{\partial}{\partial r} - \frac{\gamma_{r\phi}}{\gamma_{\phi\phi}}\frac{\partial}{\partial\phi}\right), \qquad (13)\\
\boldsymbol{i}_{\hat{\theta}} &= \frac{1}{\sqrt{\gamma_{\theta\theta}}}\frac{\partial}{\partial\theta},
\end{aligned}$$

where $\mathcal{A} = \sqrt{\gamma_{\phi\phi}\gamma_{\theta\theta}/\gamma}$. Notice, that ρ, P, $u^{\hat{i}}$ and $B^{\hat{\phi}}$ are defined at the geometrical centers of the computational cells, whereas $B^{\hat{r}}$ and $B^{\hat{\theta}}$ are defined at the geometrical centers of the cell interfaces.

At the beginning of each time step both $B^{\hat{r}}$ and $B^{\hat{\theta}}$ are found in the cell centers via linear interpolation. The cell centered values are then used to set the Riemann problems at the cell interfaces using a slope-limited interpolation. These problems are then solved using the special relativistic Riemann solver described in (Komissarov 1999), the solution being interpreted as the one observed by the FIDO initially located at the interface. In order to find the resolved state at the interface, the relative motion of the interface and its FIDO has to be taken into account (Pons et al.1998). This resolved state allows to compute the interface fluxes of all conserved quantities , first in the FIDO basis and then in the coordinate basis via the corresponding transformation laws. These fluxes are then used to evolved the volume averaged mass, energy, and momentum densities, as well as B^ϕ. The values of E_ϕ in the resolved states at the cell interfaces allow to find the mean values of E_ϕ at the cell edges. These are used to evolve the magnetic flux through the cell interfaces and, hence, to update the r- and θ-components of the interface magnetic field, first in the coordinate basis and then in the FIDO's basis via the corresponding transformation law. Next, the updated r- and $\theta-$components of magnetic field at the cell centers are found via linear interpolation. Finally, the updated conserved variable are transformed into FIDO's basis and the updated primitive variables are found using the same iterative procedure as in the special relativistic scheme. The second order accuracy in time is achieved via a half time step as described in Komissarov (1999).

The scheme was tested using the suit described in (Koide et al.1999). In all cases, where the test problem was based on an existing analytical solution, the agreement between numerical and analytical solutions is very good. However, the suite also includes the "sub-Keplerian disc" problem for which there is no analytical solution. Surprisingly, our numerical solution for this problem is dramatically different from the one of Koide et al.(1999) – instead of "bouncing of the centrifugal barrier" our disc gets swallowed by the hole. Since the specific angular momentum of this disc is lower than the one of the last stable orbit, the "bouncing" seems highly unlikely and we suspect that the results by Koide et al.(1999) are incorrect (in fact, this has been confirmed in private communication to the author by K.Shibata.) In addition, we considered the problem of an equilibrium torus around a rotating black hole, both with and without an azimuthal magnetic field. The pure gasdynamic solution is described in (Abramowicz et al. 1978). The solution for a magnetized torus does not seem to have been described in the literature (a paper is being prepared for publication elsewhere.)

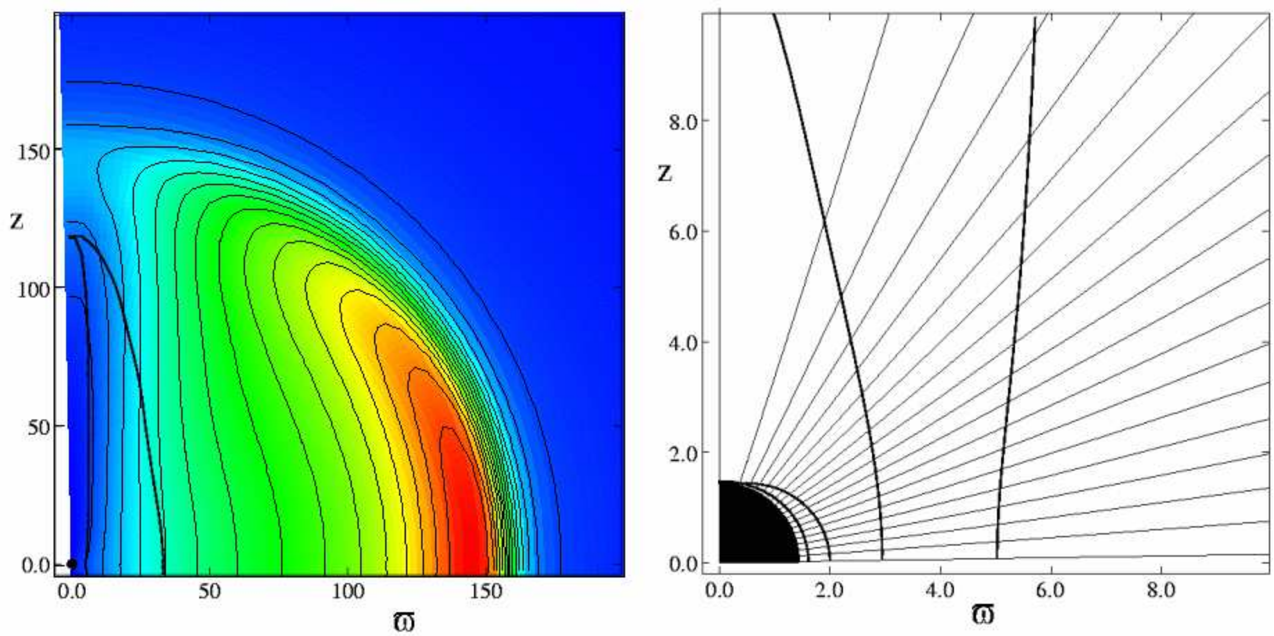

Figure 1. Solution for a black hole with $a = 0.9$ at $t = 170$. *Left panel:* Thin contour lines show the distribution of the Lorentz factor, W. There are 15 contours equally spaced between $W = 1$ and $W = 13.6$. Along the equatorial plane W is gradually increasing until it reaches a maximum at the cylindrical radius $\varpi \approx 150$. The thick lines show the Alfvén and fast magnetosonic critical surfaces of the outgoing wind. *Right panel:* The inner region of this solution. The thin lines show the magnetic flux surfaces. The thick lines show, in the order of increasing distance from the black hole, the Alfvén surface of the ingoing wind, the ergosphere, the wind separation surface, and the Alfvén critical surface of the outgoing wind. The inner fast surface is too close to the event horizon to be seen in this figure.

3 NUMERICAL SIMULATIONS

In these simulations, the computational grid has 100 cells in the θ-direction, where it is uniform, and 150 cells in the r-direction. The cell size in the r-direction is such that the corresponding physical lengths in both directions are equal. The radiative outer boundary is located far away from the event horizon, $r_{out} \approx 230$, which ensures that it does not effect the solution near the black hole.

The initial solution describes a purely radial magnetic field,

$$\boldsymbol{B} = \frac{B_0 \sin\theta}{\sqrt{\gamma}} \frac{\partial}{\partial r}, \tag{14}$$

with $B_0 = 1$ (this implies a non-vanishing azimuthal component in the Boyer-Lindquist basis.) The plasma velocity relative to FIDO is set to zero, $u^{\hat{i}} = 0$, whereas its pressure and density are set to the same value, of $p = \rho = B^2/100$. In any conservative scheme for the relativistic magnetohydrodynamics, there is an upper limit on the magnetization of plasma above which the hydrodynamical part of the solution suffers from large numerical errors (Komissarov 2001a; Gammie et al.2003). At this limit, which depends on the resolution, the numerical error for the energy density of the electromagnetic field becomes comparable with the energy density of matter. For this reason we could not set the initial density and pressure to the much lower values typical for black hole magnetospheres. Moreover, soon after the start of the simulations, the numerical solution can be described as a pair of particle winds, one ingoing and one outgoing (see figure 3), and, as the result, the plasma density gradually slides down towards the danger zone near, as well as inside, of the wind separation interface. To keep the magnetization reasonably low, we had to continuously pump new plasma in this region. The critical condition we set in these simulations was

$$wW^2 - p = 0.03B^2, \tag{15}$$

where W is the Lorentz factor of the flow as measured by FIDO. When the energy density of matter dropped below $0.03B^2$, both ρ and p were artificially increased by the same factor. To minimize the effect of the mass injection on the winds kinematics the velocity of the injected matter was set to be equal to the local velocity of the wind. In fact, new particles must be constantly created in real magnetospheres of black holes but the details of this process can be rather different (Beskin et al.1991; Hirotani & Okamoto 1998; Phinney 1982).

An additional lower limit was set on the value of the thermodynamic pressure, which was not allowed to drop below 0.01ρ. In these simulations we used the polytropic equation of state with the ratio of specific heats, $\Gamma = 4/3$.

Figure 1 shows the numerical solution for a black hole with $a = 0.9$ at $t = 170$. In the left panel of this figure, where the distribution of the Lorentz factor as measured by the local FIDO is shown, one can see an almost spherical wave front which designates the expanding boundary of the outgoing wind. This wind is ultra-relativistic and superfast within most of its volume. Moreover, the positions of both the fast and the Alfvén critical surfaces in the equatorial plane do no longer show any noticeable variation at this

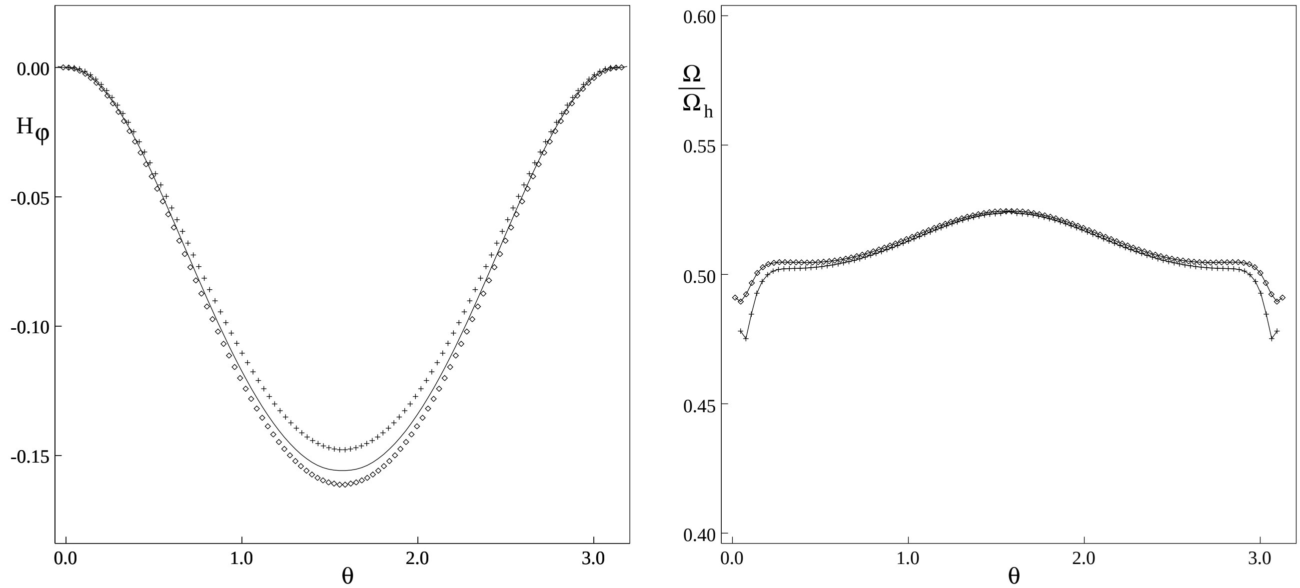

Figure 2. The angular distribution of H_ϕ and Ω for a black hole with $a = 0.9$ at $t = 170$. *Left panel:* H_ϕ; the diamonds show the MHD solution at $r = 5$ and the crosses show this solution at $r = 50$. The continuous line show the corresponding FFDE solution at $r = 5$. *Right panel:* Ω; the diamonds show the MHD solution at $r = 5$ and the crosses show this solution at $r = 50$.

point. The wind is magnetically dominated but its magnetization is slowly decreasing with distance.

While the Lorentz factor of the outgoing wind gradually increases with the distance from the black hole, and so does the inertia of the accelerated particles, the Lorentz factor of the ingoing wind remains lower than $W = 2$ all the way down to the inner boundary of the computational domain (see also figure 3). Thus, the electromagnetic field does not push the plasma of the ingoing wind onto the high-velocity orbits and its inertia does not become a key factor in the flow dynamics. On the contrary, new particle have to be constantly injected into this wind to keep its inertia above the low level given by eq.15. The region of particle injection is elongated along the symmetry axis with the major semi-axis about 20 and the minor semi-axis about 6.

The right panel of this figure shows the key surfaces in the inner part of this solution where it has settled to a steady-state. The comparison with fig.1b in (Komissarov 2001b) reveals that the locations of the Alfvén surfaces are very close to those in the corresponding FFDE solution. As expected, the surface separating the ingoing wind from the outgoing one is located between the Alfvén surfaces (Takahashi et al.1990).

Figure 2 shows the distribution of the angular velocity of magnetic field lines, Ω, and the H_ϕ-component of vector $\boldsymbol{H}$ defined via

$$H_i = \frac{\alpha}{2} e_{ijk} F^{jk}. \tag{16}$$

In steady-state

$$\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J}, \tag{17}$$

where $\boldsymbol{J}$ is the electric current density (Komissarov 2004) and, thus, H_ϕ is a measure of the poloidal current. (In Blandford & Znajek (1977) H_ϕ is denoted as B_T.) Ω and H_ϕ are very important quantities as they determine the poloidal fluxes of the electromagnetic energy and angular momentum (Blandford & Znajek 1977; Komissarov 2004). For example, the energy flux density, $\boldsymbol{S}_p$, is given by

$$\boldsymbol{S}_p = -(H_\phi \Omega) \boldsymbol{B}_p, \tag{18}$$

where $\boldsymbol{B}_p$ is the poloidal component of the magnetic field. As one can see in figure 2 the distributions of Ω and H_ϕ in our magnetically dominated MHD solution are very close to those found in the FFDE solution (Komissarov 2001b), all the way up to the fast critical surface of the outgoing wind. Thus, the particle inertia does not have much of an effect on the process of extraction of energy and angular momentum, which remains essentially electromagnetic.

Figure 3 shows the velocity distribution in the equatorial plane near the black hole. At the point separating the inflow from the outflow the angular velocity of plasma, $v^\phi = d\phi/dt$, equals to the one of the magnetic field lines. Closer to the black hole $v^\phi > \Omega$ and near the event horizon it even exceeds the angular velocity of the black hole, Ω_h. (Notice, that it is only the Boyer-Lindquist v^ϕ which always equals to Ω_h at the event horizon.)

Figure 3 also shows that the Lorentz factor of the ingoing wind remains very low and, thus, there is no any reason for the breakdown of the FFDE approximation at the event horizon suggested in (Punsly 2004; Macdonald & Thorne 1982).

4 DISCUSSION AND CONCLUSIONS

In this paper we described the first ever GRMHD simulations of monopole magnetospheres of rotating black holes. Given the central role played by the monopole problem in

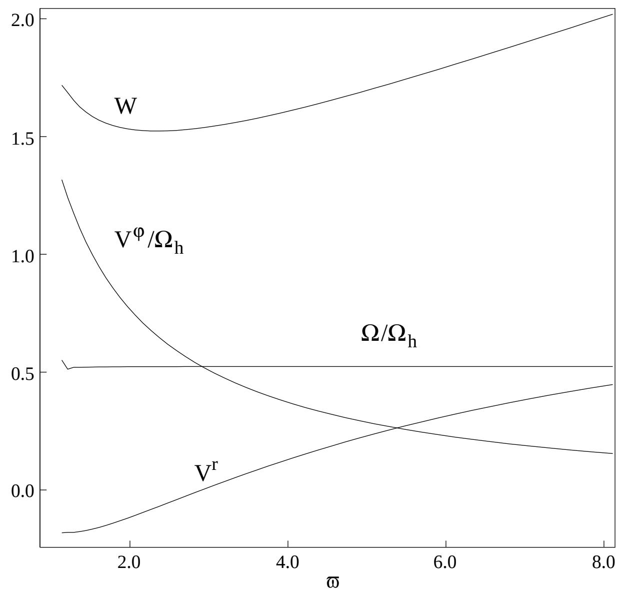

Figure 3. The Lorentz factor as measured by FIDO, W, the radial velocity of plasma, $v^r = dr/dt$, the angular velocity of plasma, $v^\phi = d\phi/dt$, and the angular velocity of magnetic field lines, Ω, in the equatorial plane at $t = 170$.

the development of the general theory of black hole magnetospheres it is not surprising that the results of these simulations allow to re-examine a number of important issues of the theory.

The main conclusion which follows from these results is the validity of the FFDE approximation at least in the case of the monopole magnetic configuration considered by Blandford and Znajek(1977). Inertia of initially rarefied magnetospheric plasma does not grow dynamically important neither in any localized regions near the black hole horizon nor in a global sense. The system of poloidal electric current and the efficiency of energy extraction are basically the same as in our GRMHD solution as in the FFDE solution found earlier (Komissarov 2001b). This means that in the magnetospheres of black holes there exists an electromotive force that has nothing to do with particle inertia. In a separate paper (Komissarov 2004) we show that, in great contrast to the Faraday disc or a magnetized stellar wind, the poloidal currents in the Blandford-Znajek model are driven by the so-called "gravitationally induced" electric field, which was first discovered by Wald (1974). Contrary to the conclusion reached in (Punsly & Coroniti 1990a), this field cannot be screened within the black hole ergosphere by any static distribution of electric charge. We also show there that Znajek's horizon condition is not a boundary condition after all, but a regularity condition imposed at the fast critical point of the ingoing wind in the limit of vanishing particle inertia. This proves the legitimacy of its utilization in the steady-state solutions by Blandford-Znajek(1977) and Phinney(1982).

In addition, the numerical solution shows a number of other interesting features that deserve discussing.

Our GRMHD solution remains very close to the FFDE one all the way up to the fast critical surface of the outgoing wind and even at the fast critical surface the wind is still Poynting flux dominated. Further away the electromagnetic energy may be transferred to the wind particles. However, the details of this energy transfer as well as the details of the interaction between the wind and its surrounding which is responsible for the observational phenomena like superluminal jets, radio galaxies etc. cannot effect the wind solution in the sub-fast region. This makes us wonder whether the key global properties of the Blandford-Znajek mechanism, such as its efficiency, are sensitive at all to the nature of the so-called "astrophysical load" (Macdonald & Thorne 1982; Thorne et al.1986).

Given the potential importance of this finding let us show that a RMHD flow may indeed stay Poynting flux dominated at the fast critical point. For this purpose we consider a one dimensional flow in flat spacetime. In the limit of cold MHD, the ratio kinetic energy flux to the Poynting flux is

$$\kappa = \rho W^2/B_t^2,$$

where B_t is the tangential component of magnetic field. At the fast point the flow velocity

$$v^2 = b^2/(b^2+\rho),$$

where $b^2 = B_n^2 + (B_t/W)^2$, B_n being the normal component of the magnetic field. From these two equations one finds that at the fast point

$$\kappa = \frac{1}{W^2} + \frac{B_n^2}{B_t^2}. \tag{19}$$

Thus, the flow is Poynting dominated provided the fast speed is ultra-relativistic and the magnetic field is predominantly tangential.

These are the first GRMHD simulations where an ultrarelativistic outflow is produced by a black hole. In all previous simulations of this sort (Koide et al.1999; Koide et al.2000; Koide 2003; Komissarov 2001a; Gammie et al.2003; De Villiers & Hawley 2003) no such high velocity outflows were reported, which was a bit worrying given the well known results of various astrophysical observations. In more realistic simulations of the future, which will include both the accretion disc and its magnetised corona, it may be possible to obtain not only ultra-relativistic but also well collimated outflows.

Since the fast and the Alfven surfaces of the outgoing wind are relatively far away from the event horizon, one may expect the outgoing wind to be more or less accurately described by the flat space monopole solution. According to (Beskin 1997) the ratio of the fast suface radius, r_f, and the Alfvén surface radius, r_a, in the equatorial plane of this solution is

$$r_f/r_a \approx \sigma^{1/3},$$

whereas the Lorentz factor at the fast surface

$$W_f = \sigma^{1/3},$$

where σ is the magnetization papameter. In our simulations $\sigma^{1/3} \approx 4.08$, $r_f/r_a = 6.60$, and $W_f = 4.35$. Thus, the numerical and the analytical results agree quite well.

The presented results also suggest a way of dealing with such more realistic problems of computational astrophysics

of black holes which involve both magnetically dominated ultrarelativistic components and particle dominated slow components. Namely, one may use the same GRMHD approximation to model all components of the system but impose an upper limit on the magnetization of the ultrarelativistic component, via appropriate floor values for both the pressure and density of matter. Although, there is nothing particularly new in such an approach and utilization of similar lower/upper limits is widely spread in computational practice, the accurate representation of magnetically dominated component in our simulations is reassuring.

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